2. E. Stewart et al., Intern. J. Heat Mass Transfer, 16, No. 2 (1973).
3. J. W. Strutt (Lord Rayleigh), Theory of Sound, 2nd ed. Dover (1937).
4. J. O. Hinze, Turbulence. An Introduction to Its Mechanism and Theory, McGraw-Hill (1959).
5. M. G. Sirotyuk, in: High-Power Ultrasonic Fields [in Russian], Nauka, Moscow (1968), p. 167.

MODEL OF HEAT TRANSFER ASSOCIATED WITH THE SURFACE BOILING OF LIQUID IN TUBES

A. F. Zaletnev

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A model is proposed for the heat-transfer process associated with the surface boiling of liquid in tubes. An analysis of the various constituents of the heat-transfer mechanism distributed over the thickness of the intermediate boiling layer is presented.

Heat transfer at various points within the thickness of the evaporating layer created by the surface boiling of liquids in tubes is made up of a number of components differing essentially in mechanism. We shall here make a further attempt at analyzing these constituents, and after allowing for their interactions we shall present a method of analyzing and generalizing experimental data regarding the heat transfer associated with the boiling of an underheated liquid.

A detailed analysis of the intensity of heat transfer in the course of boiling should allow for the effect of the conditions at the liquid—vapor—solid boundary. However, there is as yet insufficient information available to be able to formulate reliable initial data for constructing an analytical model of the heat-transfer process.

In our subsequent analysis, instead of considering the characteristics of the vaporization mechanism (number of vaporization centers, separation frequency, and separation diameter of the bubbles) separately, we shall introduce an integrated characteristic in the form of the true volumetric vapor content. In order to calculate this quantity we may, for example, make use of the empirical formula of [1] or the method recommended in [2].

Following [2-4], we distinguish two zones in the flow: the zone of wall (boundary) heating and the underheated core of the flow. Inside the boundary zone the enthalpy of the flow is greater than the saturation enthalpy of the liquid, while its boundary with the core of the flow may be found by using equations derived for calculating heat transfer during the flow of a single-phase heat carrier, i. e., on the assumption that no boiling occurs in the channels. This will give the isothermal surface in the flow corresponding to temperature t_s.

An analysis of experimental data in [2] showed that the ratio of the area of the boundary zone to the true volumetric vapor content was constant along the whole length of the channel:

$$\varphi = \varkappa\varphi', \tag{1}$$

where $\varphi' = [1-(r_c^2/R^2)]$ is the relative area of the heated zone, and $\varkappa$ is a proportionality factor depending in the following way on the operating parameters:

$$\varkappa = 30\left(\frac{q\lambda'}{\alpha_{d.b}\, r_q \mu'}\right)^{1.15} \tag{2}$$

If the true volumetric vapor content is known, the radius of the underheated core of the flow may be calculated from the equation

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TABLE 1

P, bars	$A\cdot 10^{-6}$	β
20	1,67	0,86
40	3,57	1,0
70	8,6	1,17

$$r_c = R\sqrt{1-\frac{\varphi}{\varkappa}}\,; \tag{3}$$

otherwise, the recommendations of [2] should be employed.

As regards the heat-transfer mechanism, we shall subsequently consider that part of the wall surface is washed by liquid heated to the saturation temperature, while the rest is in contact with liquid at a temperature considerably below t_s. Correspondingly, the specific thermal flux at the wall may be expressed in the form of the sum of two components:

$$q = \eta\alpha_{conv}(t_w - t_l) + (1-\eta)\,\alpha_{d.b}(t_w - t_s), \tag{4}$$

where η is the relative proportion of the channel surface washed by the underheated liquid. We consider that liquid at a temperature t_c leaks into the boundary zone, is heated by convection at the wall, and receives heat from condensing vapor bubbles. The liquid is repelled by the vapor bubbles into the core of the flow, having a temperature equal to the saturation point.

Thus, the heat carried into the core of the flow is made up of the heat collected from the wall by the convection of the underheated liquid and the heat passing into the liquid by the condensation of the vapor bubbles:

$$\Delta Q_c = \Delta Q_{conv} + \Delta Q_{cond}\,. \tag{5}$$

Let us consider the various components of heat transfer.

a) Heat Transfer into the Core of the Flow. Let us introduce the concept of the heat-transfer coefficient from the boundary zone to the core of underheated liquid α_0. The specific thermal flux through the surface with coordinate r_c is then

$$q_c = \alpha_0(t_s - t_c), \tag{6}$$

where the temperature head $(t_s - t_c)$ may be written in terms of the temperature head $(t_s - t_l)$:

$$t_s - t_c = \frac{G_0}{G_c}(1-\chi)(t_s - t_l). \tag{7}$$

Thus, the total amount of heat passing into the core of the underheated liquid may be expressed by the relationship

$$\Delta Q_c = 2\pi r_c\frac{G_0}{G_c}(1-\chi)\,\alpha_0(t_s - t_l), \tag{8}$$

and allowing for the fact that

$$G_c = G_0\frac{1-\chi}{1-\varphi}\,\frac{r_c^2}{R^2}, \tag{9}$$

we may write Eq. (7) in the following form:

$$\Delta Q_c = 2\,\frac{F}{r_c}(1-\varphi)\,\alpha_0(t_s - t_l). \tag{10}$$

If the amount of heat ΔQ_c is referred to the area of the heat-emitting surface, we obtain the effective heat-transfer coefficient to the liquid:

$$\alpha_w = \frac{R}{r_c}(1-\varphi)\,\alpha_0. \tag{11}$$

The value of the coefficient α_W may be determined if we make use of existing recommendations regarding the calculation of the true volumetric vapor content and the liquid temperature distribution along the channel.

Let us write down the heat-balance equation for the liquid phase:

$$(1-\chi)\frac{dt_l}{d\bar{z}} = \frac{\Pi z_{ch}}{\rho w F C_p}\alpha_w(t_s - t_l). \tag{12}$$

In view of the smallness of χ in the region of underheating we take

$$(1-\chi)\frac{dt_l}{d\bar{z}} \simeq \frac{dt_l}{d\bar{z}} \tag{13}$$

and integrate (12) subject to the following boundary condition:

$$\Delta t_l|_{\bar{z}=0} = \Delta t_0. \tag{14}$$

Under the conditions assumed, the solution of (12) takes the form

$$\frac{\Delta t_l}{\Delta t_0} = \exp\left[-\frac{\Pi z_{ch}}{\rho w F C_p}\int \alpha_w d\bar{z}\right], \tag{15}$$

where z_{ch} is the length of the boiling section up to the cross section $x_b = 0$.

Let us assume that the coefficient α_W may be approximated by a polynomial of degree n:

$$\alpha_w(\bar{z}) = \alpha_{NK} + \sum_{i=1}^{n} P_i \bar{z}^i, \tag{16}$$

where $\alpha_{NK} = q/\Delta t_0$.

Instead of Eq. (15), we may now write

$$\frac{\Delta t_l}{\Delta t_0} = \exp\left[-\frac{\Pi z_{ch}}{\rho w F C_p}\left((\alpha_{NK}\bar{z} + \sum_{i+1}^{n}\frac{1}{i+1}P_i\bar{z}^{i+1}\right)\right]. \tag{17}$$

The resultant equation (17) enables us to calculate the coefficients of the polynomial (16) if we know the temperature distribution of the liquid along the channel. Ultimately, we may determine the heat-transfer coefficient α_W and hence the amount of heat passing into the core of the flow.

b) Heat Evolution during the Condensation of Vapor in the Flow. By analogy with [5, 6] we introduce the concept of the condensation coefficient k (W/m · °K):

$$\Delta Q_{cond} = k(t_s - t_l) \tag{18}$$

and establish a relationship between this coefficient and α_0, Δt_l.

In conformity with the foregoing concepts as to the mechanism of heat transfer, we write the heat balance at the wall in the form

$$q = mr_q + \alpha_w(t_s - t_l) - \frac{1}{\Pi}k(t_s - t_l), \tag{19}$$

where m is the mass of the vapor generated from unit surface of the channel. The second term corresponds to the heat carried away into the core of the flow.

In order to determine the condensation coefficient k we must know m. Let us consider that the volume of vapor generated at the heating surface equals the volume of the radial flow of underheated liquid leaking into the boiling layer at the boundary. In this case

$$m = \frac{\alpha_w(t_s - t_l)}{C_p(t_s - t_c)}\frac{\rho''}{\rho'}. \tag{20}$$

Using Eq. (19) with due allowance for (17) and (20), we obtain a relationship for the condensation coefficient:

$$k = \Pi\left\{\alpha_w\left[\frac{\rho'' r_q}{\rho' C_p(t_s - t_c)} + 1\right] - \frac{q}{t_s - t_l}\right\}. \tag{21}$$

c) Calculation of the Amount of Heat Carried away from the Wall by the Convection of Underheated Liquid. It follows from Eq. (5) that

$$\Delta Q_{conv} = \Delta Q_c - \Delta Q_{cond} \tag{22}$$

and on allowing for (10) and (18), we may write

$$\Delta Q_{conv} = 2\frac{F}{r_c}(1-\varphi)\alpha_0(t_s - t_l) - k(t_s - t_l). \tag{23}$$

All the quantities on the right-hand side of Eq. (23) have already been determined; thus, within the framework of the present analysis we may consider that we have established ΔQ_{conv}.

Let us turn to Eq. (4) and write down the following extra equation:

$$(1-\eta)\Pi\alpha_{d.b}(t_w - t_s) = \Pi q - \Delta Q_{conv}. \tag{24}$$

From Eqs. (4) and (24) we find an equation for the convective heat-transfer coefficient from the wall to the underheated liquid:

$$\alpha_{conv} = \frac{\alpha_{d.b}}{\frac{\Pi\alpha_{d.b}(t_w - t_{wd.b})}{\Delta Q_{conv}} + 1}\,\frac{t_w - t_s}{t_w - t_l}. \tag{25}$$

Thus, the problem of calculating the coefficient α_{conv} may be regarded as ended. However, in so considering we assume that the liquid temperature encountered in the analysis may be determined experimentally or calculated from existing empirical formulas [7].

The main assumption in the foregoing calculation lies in the expansion (4).

We executed a numerical calculation of α_{conv}, taking the true volumetric vapor content of the channel from the empirical equation of [1], while in order to determine the temperature of the liquid t_l we used the equation for calculating slip in a flow introduced in [6]:

$$s = \left(\frac{\rho'}{\rho''}\right)^{0.205} \mathrm{Re}^{-0.016}, \tag{26}$$

where $\mathrm{Re} = \rho w 2R/\mu'$.

After we have found the quantities φ and s, the temperature of the liquid may be obtained from the equation

$$t_b = \frac{\left(t_s + \frac{r_q}{C_p}\right)\chi - t_l}{\chi - 1}, \tag{27}$$

where

$$\chi = \frac{s\varphi\rho''}{s\varphi\rho'' + (1-\varphi)\rho'}; \quad t_b = \frac{r_q x_b}{C_p} + t_s. \tag{28}$$

The thermal state of the channel wall was calculated as recommended in [8, 9].

As a result of this it was found that the convective heat-transfer coefficient of the underheated liquid in the range of operating parameters for which the α_{conv} formula [1] remained valid could be expressed as the following power relationship:

$$\alpha_{conv} = A w_g^{\beta}, \tag{29}$$

where

$$w_g = m/\rho''; \tag{30}$$

A and β are numerical quantities depending on the pressure. In particular, for different pressures our calculations give the values shown in Table 1.

The proposed method of analyzing heat transfer in a flow of boiling heat carrier may be used for calculating the temperature and thermal-flux distributions along the boiling economizer in a steam generator (boiler) of the heating-liquid [wall] boiling-liquid type. For this purpose the section of channel under consideration should be nominally divided into individual parts and the thermal state calculated for each by the iteration method.

NOTATION

z, coordinate in the direction of flow (distance from initial point of vapor generation); z_{ch}, length of channel section from the initial point of vapor generation to the cross section in which the temperature of the flow equals the saturation temperature; $\bar{z} = z/z_{ch}$, relative axial coordinate; R, radius of channel; Π, heated perimeter; r_c, radius of the core of the flow; φ, true volumetric vapor content; χ, true gravimetric vapor content; x_b, balance vapor content; P, pressure; r_q, latent heat of vaporization; q, thermal flux; $\alpha_{d.b.}$, heat-transfer coefficient for developed bubble boiling; α_{conv}, convective heat-transfer coefficient for underheating of the flow; λ', thermal conductivity of the liquid; C_p, specific heat of the liquid at constant pressure; ρ', density of the liquid; ρ'', density of the vapor; μ', dynamic viscosity of the liquid; t_s, saturation temperature; t_w, wall temperature; $t_{w.d.b.}$, wall temperature for developed bubble boiling; t_c, average temperature of the liquid core; Δt_l, underheating of the liquid (difference between the liquid saturation temperature and the temperature of the liquid in the cross section z); Δt_0, underheating of the liquid at the initial point of vapor generation; G_0, mass rate of flow; G_c, mass rate of flow in the core; F, cross-sectional area of the channel; ρw, mass velocity; w_0, circulation velocity; s, slip (ratio of the velocity of the vapor to the velocity of the liquid); k, condensation coefficient; ΔQ_c, amount of heat transferred to the liquid core per unit time per unit length of channel; ΔQ_{conv}, amount of heat transferred as a result of the convection of the underheated liquid per unit time per unit length of channel; ΔQ_{cond}, amount of heat passing to the underheated liquid as a result of the condensation of bubbles per unit time per unit length of channel.

LITERATURE CITED

1. Z. L. Miropol'skii, R. I. Shneyerova, and A. I. Karamysheva, in: Proceedings of the Fourth International Heat Transfer Conference, Paris (1970).
2. E. A. Zakharova, B. A. Kolchugin, I. P. Kornyukin, D. A. Labuntsov, and A. G. Lobachiov, in: Proceedings of the Fourth International Heat Transfer Conference, Paris (1970).
3. S. S. Kutateladze, Heat Transfer in Condensation and Boiling [in Russian], Mashgiz, Moscow—Leningrad (1952).
4. E. I. Nevstrueva and Kh. M. Gonsales, Teploénergetika, No. 9, 34 (1960).
5. Z. Ruani, Teploperedacha, Ser. C, No. 3, 117 (1968).
6. S. Akhmad, Teploperedacha, Ser. C, No. 4, 27 (1970).
7. N. Zuber and F. W. Staub, in: Advances in Heat Transfer [Russian translation], Mir, Moscow (1970).
8. Yu. E. Pokhvalov, Author's Abstract of Candidate's Dissertation, Moscow Physical Engineering Institute, Moscow (1964).
9. V. M. Borishanskii, Transactions of the Central Boiler and Turbine Institute [in Russian], Vol. 108 (1971), p. 25.